International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics
1995 - Third International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

05 Apr 1995, 1:30 pm - 3:30 pm

Elastic Models of Rock Due to Explosion Pressures

Sanga Tangchawal
Chulalongkorn University, Bangkok, Thailand

Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics
April 2-7, 1995; Volume III, St. Louis, Missouri

Elastic Models of Rock Due to Explosion Pressures

Paper No. 10.15

Sanga Tangchawal
Chulalongkorn University
Bangkok, Thailand

SYNOPSIS Basic theories of wave pulse initiated from conventional blasting were reviewed. Mechanics of various stress wave forms of explosion pressure were proved. A new suggested solution of displacement potential by Rinehart instead of Sharpe or Duvall's suggestion, was used to find the optimum solution. From the mathematical research models, in which they were written as a package program in C language and used Microsoft visual basic as user interface, they proved that a typical cylindrical charge explosion in a bench may take on some of the characteristics of a concentrated charge explosion.

1 INTRODUCTION

Nature of explosion wave front is nonplanar and its shape is either spherical or cylindrical wave pulse. Conventional models suggested by Sharpe and those of Duvall are found useful in the range in which the values of Lame's constant and modulus of rigidity are equal. To extend the findings, the mathematical model using a displacement potential value suggested by Rinehart is better than those of conventional models due to the solution is more practical to be used for any competent rocks that usually have the value of Lame's constant does not equal to the value of modulus of rigidity.

The partial derivatives for Rinehart's displacement potential function with respect to distance (and also time) are applied to find the elastic dynamic parameter values such as displacement, particle velocity, acceleration, etc. Similar to Duvall's method, the researcher uses the method of superposition in order to have a double pulse form for exponential and step function attenuation. The derived wave equations are compared and analyzed using field measurement values and results are acceptible within the elasticity limit.

2 OBJECTIVES OF STUDY

This research paper presents a review of the past distinguished works on simulation for explosion wave pulses and to develop specific concepts of the following items:

1. Wave equations in a three - dimensional form
2. Derivation of analytical results for a long cylindrical charge
3. Set computational methods for three - dimensional explosion wave model of bench blasting and written a package program in C language.

3 OVERVIEW OF FRAGMENTATION AND MODELLING

In modelling long cylindrical charges for bench blasting, the shape of the pressure pulse applied to the cavity wall by the explosive or the shape of the strain wave in the rock have to be measured experimentally or assumed. Since the explosive used in blasting is always loaded into cylindrical blastholes, the simplest method of modelling cylindrical charges is to assume that the axial strain can be ignored and that the initial pressure is the explosion pressure.

Sharpe has determined theoretically the wave motion resulting from the application of a pressure pulse to the interior of a spherical cavity in an elastic rock medium. His solution is based upon the assumptions that Poisson's ratio is 0.25 (or a Lame's constant equals to modulus of rigidity), and that the pressure is applied normal to the spherical surface. The U.S. Bureau of Mines studied the generation of strain waves produced by the detonation of high velocity explosives placed in small cavities in the rock (Obert and Duvall, 1949; Duvall and Atchison, 1950; Duvall, 1953; Duvall et al., 1957; Duvall et al., 1967). Conventional solution of displacement potential for spherical expanding waves was developed by Duvall using the basic idea of Sharpe's solution. The application of a pressure pulse of the double exponential decay pulse form

$$P(t) = P_0\left(e^{-\alpha t} - e^{-\beta t}\right) \quad \text{for } \beta > \alpha \tag{1}$$

where α and β are constant of pressure pulse parameters

would be given the displacement potential resulting from the method of superposition by

$$\phi = \frac{aP_0 / \rho\gamma}{\left(\frac{\omega}{\sqrt{2}} - \alpha\right)^2 + \omega^2} \cdot \left\{-e^{-\alpha\tau} + e^{-\omega\tau/\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}} - \frac{\alpha}{\omega}\right)\sin\omega\tau + \cos\omega\tau\right]\right\} \tag{2}$$

$$+ \frac{aP_0 / \rho\gamma}{\left(\frac{\omega}{\sqrt{2}} - \beta\right)^2 + \omega^2} \cdot \left\{-e^{-\beta\tau} + e^{-\omega\tau/\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}} - \frac{\beta}{\omega}\right)\sin\omega\tau + \cos\omega\tau\right]\right\}$$

$$\text{for } \tau \geq 0$$

$$= 0 \quad \text{for } \tau < 0$$

When one takes the partial derivatives of displacement potential with respect to distance and time and arranged in the appropriate forms, they will be analytical results for those elastic variables due to an explosion pressure such as displacement, particle velocity, particle acceleration, radial stress and strain, tangential stress and strain.

No analytical results for cylindrical charges similar to those for spherical charges are successfully developed and known to the author. Numerical results by Selberg (1952) and confirmed by Harries (1983) showed that the dominant frequency produced by a cylindrical charge was markedly lower than the frequency which would be expected from a spherical charge of the same diameter as the cylinder. Comparing the calculated frequency from a sphere to that for a cylinder showed that the frequency from a cylinder was 1/6th of that from a sphere. Selberg also introduced the LaPlace transform, complex function, and an asymptotic expression in order to solve the closed form of cylindrical wave equation. Selberg concluded that in static equilibrium the stresses decrease in proportion to $1/r^2$ with increasing a radial distance, while the stresses in the dynamic wave front decrease in proportion to $1/r^{1/2}$. Aso (1966) found similar results as those results by Selberg. Jordan (1962) derived the wave propagation equations from a finite, cylindrical explosive source. He suggested that the rate of attenuation would be $1/r^{1/2}$ for those waves closer to the ignition source and would be $1/r$ for those waves of a large radial distance.

4 DEVELOPMENT TO CYLINDRICAL CHARGE MODELS

An understanding of the generation and propagation of the stress around long cylindrical charges has made progress rather slow, one of techniques for recording strain pulses (Plewman and Starfield, 1965; Duvall et al., 1967; Starfield and Pulgliese, 1968; Dawes, et al., 1983; Swoboda and Ning Li, 1993) confirms that the gradual build up to peak amplitude can be summed to produce a pulse similar to that generated by the long cylindrical charge.

4.1 Modelling for a cylindrical blashole

Consider a cylindrical hole with radius "a" in an infinite elastic medium (Figure 1). The medium is at rest at the time $t = 0$, and after this time a uniformly distributed normal pressure, $P(t)$, is suddenly applied on the boundary surface of the cavity.

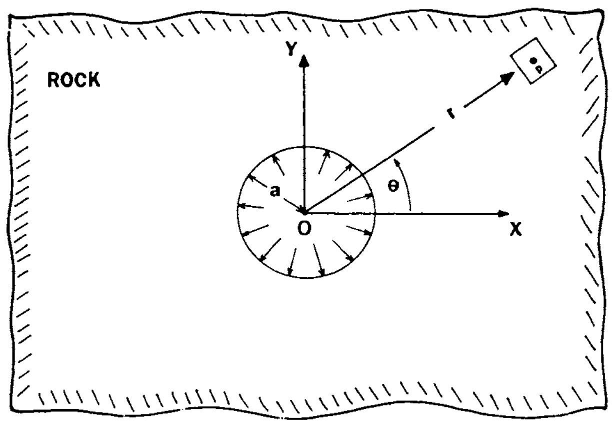

Figure 1. Idealized model for a coordinate system of stress wave from one cylindrical hole.

4.2 Computation methods for governing equations

Modelling of blasting simulation in this study has developed by derivation of Rinehart's displacement potential solution for a diverging wave from the spherical cavity. His solution can be extended to any values of Poisson's ratio and would be

$$\phi = \frac{aP_0}{\eta^2 \rho r} \cdot \left\{-e^{-\alpha\tau} + \frac{\eta}{\omega_0} \cdot e^{-\chi\tau} \cdot \cos\left(\omega_0\tau - \zeta\right)\right\} \tag{3}$$

The various constants, χ, η, ω_0 and ξ, depend upon the material, the radius of cavity, and the shape of applied pressure pulse. These constants are given by

$$\chi = (c/a)(1-2\nu)/(1-\nu) \tag{4}$$

$$\eta^2 = \omega_0^2 + (\chi - \alpha)^2 \tag{5}$$

$$\omega_0 = (c/a)(1-2\nu)^{\frac{1}{2}}/(1-\nu) \tag{6}$$

$$\zeta = \tan^{-1}\left[(\chi-\alpha)/\omega_0\right] \tag{7}$$

Other constants and relationships are similar to convention solution suggested by Duvall. These are

$$P(t) = P_0\, e^{-\alpha\tau} \tag{8}$$

$$\tau = t - \frac{r-a}{c} \tag{9}$$

$$c = \sqrt{\frac{3G}{\rho}} \tag{10}$$

$$\alpha = \frac{n\,\omega_0}{\sqrt{2}} \tag{11}$$

Appropriate differentiation and utilization of the above relationships yield expressions for radial displacement (u), particle velocity (v), particle acceleration (a) and dilatation (Δ). Following expressions, in which the author derived, have been arranged in a form suitable for computer calculations.

$$u = \frac{aP_0}{\eta^2\rho r}\cdot\left\{\left(\frac{1}{r}-\frac{\alpha}{c}\right)\cdot e^{-\alpha\tau} + \frac{\eta e^{-\chi\tau}}{\omega_0}\cdot\left[\left(\frac{\chi}{c}-\frac{1}{r}\right)\cdot\cos\left(\omega_0\tau-\zeta\right) + \frac{\omega_0}{c}\cdot\sin\left(\omega_0\tau-\zeta\right)\right]\right\} \tag{12}$$

$$v = \frac{aP_0}{\eta^2\rho r}\cdot\left\{\alpha\left(\frac{\alpha}{c}-\frac{1}{r}\right)\cdot e^{-\alpha\tau} + \eta e^{-\chi\tau}\cdot\left[\left(\frac{\omega_0^2-\chi^2}{c\omega_0}+\frac{\chi}{r\omega_0}\right)\cdot\cos\left(\omega_0\tau-\zeta\right) + \left(\frac{1}{r}-\frac{2\chi}{r}\right)\cdot\sin\left(\omega_0\tau-\zeta\right)\right]\right\} \tag{13}$$

$$\varepsilon_r = \frac{aP_0}{\eta^2\rho r}\cdot\left\{\left(\frac{2\alpha}{rc}-\frac{2}{r^2}-\frac{\alpha^2}{c^2}\right)\cdot e^{-\alpha\tau} + \eta\cdot e^{-\chi\tau}\cdot\left[\left(\frac{\chi^2-\omega_0^2}{c^2\omega_0}+\frac{2}{r\omega_0}\left(\frac{1}{r}-\frac{\chi}{c}\right)\right)\cdot\cos\left(\omega_0\tau-\zeta\right) + \frac{2}{c}\left(\frac{\chi}{c}-\frac{1}{r}\right)\cdot\sin\left(\omega_0\tau-\zeta\right)\right]\right\} \tag{14}$$

$$a = \frac{aP_0}{\eta^2\rho r}\cdot\left\{\alpha^2\left(\frac{1}{r}-\frac{\alpha}{c}\right)\cdot e^{-\alpha\tau} + \eta e^{-\chi\tau}\cdot\left[\left(\left(\chi^2-3\omega_0^2\right)\frac{\chi}{c\omega_0} + \frac{\left(\omega_0^2-\chi^2\right)}{r\,\omega_0}\right)\cdot\cos\left(\omega_0\tau-\zeta\right) + \left(\frac{\left(3\chi^2-\omega_0^2\right)}{c}-\frac{2\chi}{r}\right)\cdot\sin\left(\omega_0\tau-\zeta\right)\right]\right\} \tag{15}$$

$$\Delta = \frac{aP_0}{\eta^2\rho r}\cdot\left\{-\frac{\alpha^2}{c^2}e^{-\alpha\tau} + \frac{\eta e^{-\chi\tau}}{c^2}\cdot\left[\left(\frac{\chi^2-\omega_0^2}{\omega_0}\right)\cdot\cos\left(\omega_0\tau-\zeta\right) + 2\chi\sin\left(\omega_0\tau-\zeta\right)\right]\right\} \tag{16}$$

In order to find the other elastic variable expressions, one can use the following equations for tangential strain (ε_θ), radial stress (σ_θ) and tangential stress (σ_r).

$$\varepsilon_\theta = \frac{u}{r} \tag{17}$$

$$\sigma_r = \lambda\Delta + 2G\varepsilon_r \tag{18}$$

$$\sigma_\theta = \lambda\Delta + 2G\varepsilon_\theta \tag{19}$$

The displacement potential given in equation 3, however, is only suitable for a single exponential decay pulse. To be able to calculate such double exponential decay pulse, the method of superposition has been used and will have similar results as discussed above. The unit form of step function pulse that has the form

$$P(t) = P_0 \tag{20}$$

can be derived for their elastic variables directly from equations 12-16 by assuming the positive constant-α is zero.

4.3 Summation of three-dimensional stresses produced

Ideas of summation detonation pressures (or stresses) for a long, cylindrical charge are based on those models of detonation and wave propagation by Starfield and Pulgliese (1968), Harries (1983), and Dowding and Aimone (1985). These distinguished researchers suggested that a cylindrical charge is divided discretely into a set of charge segments, each of which is represented by an equivalent spherical charge.

5. RESULTS AND DISCUSSION

To check the hypothesis of Rinehart's displacement potential function, the author uses the Poisson's ratio of 0.25 and executed the calculation programs on both Rinehart and Duvall's concept. Analytical results for elastic dynamic variables are compared and the accuracy is within 1% errors. Though this method of general spherical wave equations has been accepted. Furthermore, the Poisson's ratio is assumed to be 0.30, 0.35, and so on. The calculation results still correlate well with field observation data.

From equations derived, the extent to which the predictions of elastic waves by explosion pressures has been developed agree qualitatively with the empirical observations. The package computer program in which it can demonstrate relationships between dependent variables (displacement, velocity, radial stress, tangential stress, etc.) with an independent variable ($\omega\tau$ - oscillatory function, or ωt-time scale function). Figure 2 illustrates the nature of displacement at distances from the center of cavity of a, 2a, 5a and 8a. Figure 3 illustrates the radial stress wave shape at infinity.

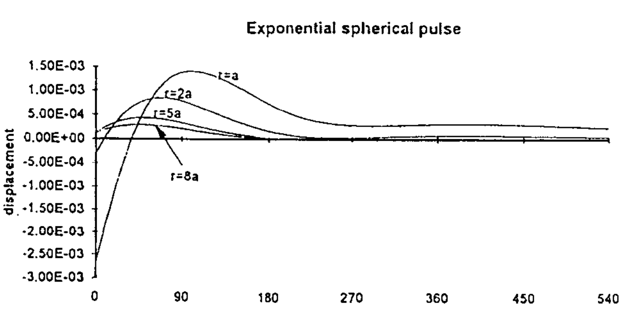

Figure 2. Relationships for exponential spherical pulse between displacement and $\omega\tau$ at varied distances.

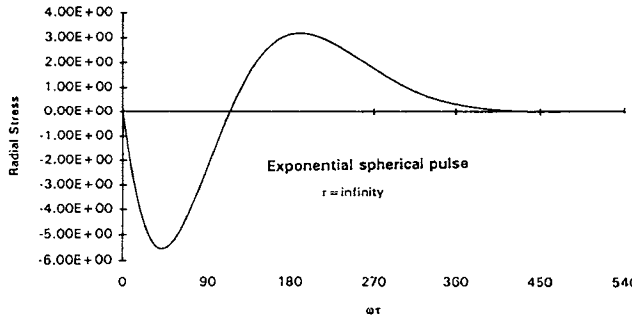

Figure 3. Correlation on exponential spherical pulse between radial stress and $\omega\tau$ at infinity.

Summation of stresses for a cylindrical wave front has been calculated by assuming that the length of charges is N spheres and having M spherical charges/source. Spherical divergent waves from each significant point sources are calculated at specified nodes beginning with the detonation time for the closest blasthole (Figure 4). The three principal stresses (one radial and two tangential) associated with each point source are rotated at the node from a rectangular coordinate system to a global coordinate system, added and stored in the computer code program.

Once the stress components are summed over time, both the eigen-vectors (the three orthogonal planes along which shear stresses vanish), and the eigen-values (the principal or normal stresses acting on these planes) are computed with a polynomial solving routine. At each time step, the principal stresses can be compared with previous values to identify the maximum.

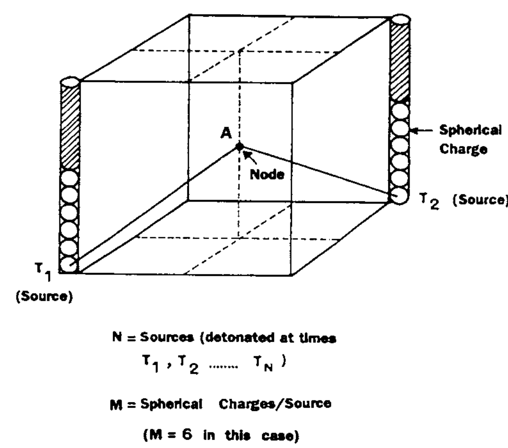

Figure 4. Geometrical relationship between a receiver node and separately exploded charges.

6 CONCLUSION

This research project introduces new analytical results for spherical transient pulse that can apply to any values of Poisson's ratio. Extending of spherical wave nature has been applied to simulate a long cylindrical charge. Based on various researchers, a cylindrical charge in the blasthole is assumed to be equal to 6 spherical wave fronts.

Summation of stresses explosively by long, multiple blastholes within a three-dimensional rock mass is computed for each node once all significant blastholes have detonated. The principal stresses are calculated at each time step with a polynomial solving routine to identify their peak value.

ACKNOWLEDGEMENTS

The author would like to thank the Faculty of Engineering, Chulalongkorn University, Thailand for supporting the expense during carried out this research. Also he is also indebted to his colleague especially, Mr. J. Lee. and Mr. P. Sriwallop for helping in equation derivation and programming.

APPENDIX : SYMBOL AND NOTATION

A = receiver node

a = radius of blasthole

α = positive time decay constant

β = second positive time decay constant

c = longitudinal (dilatation) propagational wave velocity

c_d = detonation wave velocity

c_p = primary (longitudinal) wave velocity

Δ = dilatation (change in volumes per unit volume)

E = Young's modulus

ε_r = radial strain

ε_θ = tangential strain

e = base of the natural system of logarithms

η = constant for Rinehart's function

G = modulus of rigidity

λ = Lame's constant

M = number of spherical charges/source

m = constant of the second exponential pulse

N = number of charge sources

n = a variable for integration, and also as constant of the first exponential pulse

ν = Poisson's ratio

ω = angular frequency for circular oscillation

ω_0 = angular frequency for Rinehart's function

$P(t)$ = variable (applied) pressure

P_0 = constant (detonation) pressure

ϕ = displacement potential function for spherical wave

r = radial distance from center of cavity (blasthole)

ρ = density of material

σ_r = radial stress

σ_θ = tangential stress

$T_1..T_N$ = Detonation time of exploded charges

t = time (real)

τ = time (retard), also a variable for integration pulse

θ = phase angle in cylindrical coordinate system

θ_1 = phase angle for the first exponential pulse

θ_2 = phase angle for the second exponential pulse

θ_3 = phase angle for spherical pulse

u = radial particle displacement

v = radial particle velocity

X or x = axis in cartesian coordinate system

Y or y = axis in cartesian coordinate system

Z or z = axis in cartesian coordinate system

ζ = constant for Rinehart's function

REFERENCES

Aimone, C. T. (1982), "Three-Dimensional Wave Propagation Model of Full-Scale Rock Fragmentation." Ph.D. Dissertation, Northwestern University.

Aso, K. (1966), "Phenomena Involved in Pre-Splitting by Blasting." Ph.D. Dissertation, Stanford University.

Dawes, J.J., C.K. McKenzie, T.J. Liddy (1983), Interaction between Blast Design Variables; Experimental and Modelling Studies." First International Symposium on Rock Fragmentation by Blasting, August 1983, Vol. 1, pp. 265-287.

Dowding, C. H. and C. T. Aimone (1985) "Multiple Blast-Hole Stresses and Measured Fragmentation," Vol. 18, pp. 17-36.

Duvall, W.I. (1953), "Strain-Wave Shapes in Rocks near Explosions." Geophysics Journal, Vol. 18, pp. 310-323.

Duvall, W.I. and T.C. Atchison (1950) "Vibrations Associated with a Spherical Cavity in an Elastic Medium." U.S. Bureau of Mines-Report of Investigations, No. 4692.

Duvall, W.I. T.C. Atchison (1957) "Rock Breakage by Explosives." U.S. Bureau of Mines-Report of Investigations, No. 5356.

Duvall, W.I. T.C. Atchison, and D.E. Fogelson (1967) "Empirical Approach to Problems in Blasting Research." Proceedings of the Eight Symposium on Rock Mechanics, September 1966, pp. 500-523.

Harries, G. (1983) "The Modelling of Long Cylindrical Charges of Explosive." First International Symposium on Rock Fragmentation by Blasting, August 1983, Vol. 1, pp. 419-431.

Jordan, D.W. (1962) "The Stress Wave from a Finite, Cylindrical Explosive Source." Journal of Mathematics and Mechanics, Vol. 11, pp. 503-551.

Obert, L., W.I. Duvall, B.F. Grant, R.L. Rough, and T.C. Atchison (1949) "Use of Explosives in Oil and Gas Wells-1949 Test Results." U.S. Bureau of Mines-Report of Investigations, No. 4714.

Plewman, R.P. and A.M. Starfield (1965) "The Effects of Finite Velocities of Detonation and Propagation on the Strain Pulses Induced in Rock by Linear Charges, Journal of S. African Inst. of Min. and Met., Vol. 66, pp. 77-96.

Rinehart, J.S. (1975) "Stress Transients in Solids." HyperDynamics.

Selberg, H.L. (1952) "Transient Compression Waves from Spherical and Cylindrical Cavities." Arkiv För Fysik Journal, Vol. 5, pp. 97-108.

Sharpe, J.A. (1942) "The Production of Elastic Waves by Explosion Presures. 1. Theory and Empirical Field Observations." Geophysics Journal, Vol. 7, pp. 144-154.

Starfield, A.M. and J.M. Pugliese (1968) "Compression Waves Generated in Rock by Cylindrical Explosive Charges: A Comparison between Computer Model and Field Measurements." Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., Vol. 5, pp. 65-77.

Swoboda, G. and Ning Li (1993) "Numerical Modelling of Blast Loading." Proceedings of the Fourth International Symposium on Rock Fragmentation by Blasting, July 1993, pp. 25-31.

Tangchawal, S. (1995) Research Report Work entitled "Elastic Models of Rock due to Explosion Pressures." Research Grant No. 130-GER-2536, Faculty of Engineering, Chulalongkorn University, Thailand.